Exploring parent-child interactions by learning mathematics: Repertoires-in-Use within a kindergarten-family learning environment

Ángela Uribe[1], Priska Sprenger[2] and Barbara Ott[1]

[1]St.Gallen University of Teacher Education, CH; angela.uribe@phsg.ch, barbara.ott@phsg.ch

[2]University of Education Freiburg, DE; priska.sprenger@ph-freiburg.de

This paper explores the interactions between Lami, a child fluent in Spanish and learning German and Swiss German, and his father, who moved to Switzerland three years ago. They engage in mathematical activities at home, introduced through videos. These activities are part of a broader learning environment that includes similar activities at kindergarten, conducted in the language of instruction. The paper examines how Lami and his father use their linguistic repertoires in the learning process. It provides a detailed analysis of the 'sources of meaning' used during their interactions, highlighting the diversity of repertoires, even within a single language. It reveals that the richness of language in mathematical interactions extends beyond the specific languages spoken, encompassing the full spectrum of an individual's linguistic and experiential repertoire.

Keywords: Multilingualism, kindergarten, learning at home, 'repertoires-in-use'

Introduction

Family engagement plays a crucial role in developing a multilingual environment that supports cognitive and language development in children (Cummins, 2000). Understanding the learning processes of multilingual children can be significantly enhanced by examining the interactions between parents and children during mathematical activities. These interactions often uncover a variety of cultural experiences and alternative mathematical approaches, thereby broadening the learning spectrum (Civil & Quintos, 2022).

In Switzerland, where this study is conducted, a considerable number of children are exposed to multiple languages at home: 33% of children under 15 years old encounter two languages, and 10% are exposed to three or more languages (BFS, 2021). Considering that kindergarten is part of the educational trajectory in Switzerland, where children typically attend from the age of four for two years, understanding how to coordinate family and school efforts becomes essential. Particularly, this coordination is crucial for effectively integrating aspects of multilingualism and parental involvement to support the learning process and to optimize the potential benefits of multilingualism from an early age (Baker, 2014).

In this design research study, we design a learning environment that bridges family and school contexts (Ott et al., 2024). We develop tasks for both settings that focus on developing an understanding of perceiving and using structures to determine the cardinality of sets through discussions about structured arrangements, using materials such as plastic eggs and egg cartons. Our aim is to engage the full linguistic repertoire of students through mathematically rich, playful activities. In this paper, the focus is placed on the case of Lami, a multilingual child whose linguistic repertoire includes Spanish, German, and Swiss German. We reconstruct parent-child interactions

with focus on naturally occurring argumentative situations during joint mathematical activities with materials. Furthermore, we use the approach of ‘repertoires-in-use’ to disentangle the resources used for meaning-making in these interactions.

Theoretical background

Argumentation in family interactions

Argumentation plays a crucial role not only in fostering multilingualism by promoting active, diverse language use (Moschkovich, 2002) but also in developing mathematical understanding. Beyond formal educational settings, argumentation is also deeply embedded in family dynamics, serving as a medium through which members negotiate and navigate daily challenges and differing viewpoints (Arcidiacono et al., 2022). Arguments are a natural part of family life, shaped by the relationships and dynamics within the family, and serve as both a ‘vehicle and site of socialization’ (Heller, 2023). Of our particular interest is to reconstruct how these naturally occurring argumentations unfold within the context of learning mathematics together at home.

‘Repertoires-in-Use’ for meaning-making

Different languages, dialects, and even language-related experiences can serve as ‘sources of meaning’ (Barwell, 2018) that can be activated to support the process of meaning-making. The sources of meaning that an individual has at their disposal may be difficult to reconstruct due to their complexity. This paper resonates with the concept of ‘repertoires-in-use’, which refers to the situational reconstruction of the use of one’s own repertoire for a specific purpose, specifically for making meaning by learning mathematics (Uribe & Prediger, 2021). The term ‘repertoire’, as defined by Lüdi (2016), refers to the full set of linguistic resources – encompassing languages, dialects, styles, and registers – that an individual has access to and employs to navigate different communicative contexts effectively. The conceptualization of ‘repertoires-in-use’ focuses on analyzing language use in multilingual mathematics learning, distinguishing between formal-related, meaning-related, and everyday language across activated languages. It also assesses graphical, symbolic, and contextual representations and examines how these resources are interconnected in the learning process.

To better understand the ‘repertoires-in-use’ for meaning-making, it is crucial to examine specific learning contents. Numerous studies have shown that the ability to perceiving and using structures to determine the cardinality of a set is the basis for successful arithmetical learning (e.g. Lüken, 2012; Mulligan & Mitchelmore, 2013). The study presented here focuses on such structuring processes as a fundamental aspect of mathematical learning (Sprenger & Benz, 2020).

Methodology of the design research study

The data presented in this paper are part of the project “MATHEsprechen in kindergarten: Using language repertoires in mathematics productively right from the start”, which investigates the overarching question of the extent to which the use of multilingual resources is stimulated in kindergarten children through mathematical activities with materials in a learning environment. This paper investigates the following sub-question: Which sources from their repertoire do a multilingual child and their father use during playful family interactions focused on structuring processes, and how are these sources interconnected by the child and father during the learning process?

In Design Research methodology, we developed an initial design and investigated the research question, focusing on iterative refinement through two cycles of design, implementation, analysis, and redesign, aiming to both understand and improve the learning environment and theories.

Design approach

The designed learning environment consists of activities for learning the mathematical concept of perceiving and using structures to determine cardinality at kindergarten and in the family. Although the kindergarten setting is monolingual in the language of instruction and the family setting is monolingual or multilingual in the home language(s), depending on typical family communication patterns, the study aims to bridge these contexts to enhance learning and to identify which multilingual resources activate children as 'sources of meaning' by learning mathematics. For the family setting, videos were developed to guide the activities. For each activity one video provides parents with informational content, including suggestions such as potential questions to ask their children to stimulate discussion about structuring and other video is for parents to use with the children, explaining the activities through a narrator and demonstrating with two puppets acting as language models.

Methods of data gathering and data analysis

The learning environment was tested in a design experiment in September 2023 in two kindergarten groups in Switzerland with children in the last year of kindergarten. The first group consisted of 10 children (average age: 5 years, 7 months), the second of six children (average age: 5 years, 3 months). Two Albanian-speaking families, one Spanish-speaking family and one Bosnian-speaking family took part in the family part. The whole design experiment was videotaped.

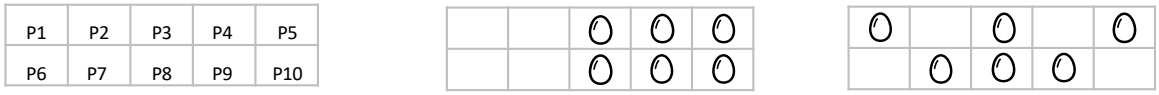
Individual scenes, chosen for their direct relevance to the research question due to the meaningful interactions occurring between children and parents, were analyzed using the interpretative methodology of interaction analysis (Krummheuer & Naujok, 1999).

Empirical insights into Lami's process

The analysis provides insights into the 'repertoires-in-use' of Lami and his father while learning mathematics oriented by the designed videos within the family setting. Lami, who is 5 years and 3 months old, moved to Switzerland three years ago. Both of Lami's parents come from Spain. His father, with whom he conducts the activities, speaks Spanish and has limited understanding of German. The home language is Spanish. The transcript displays the original utterances along with their translations in parentheses. In the analysis, only the translations are shown. The use of gray coloring for these translations indicates that the original utterances were made in a different language.

Learning mathematics together at home with videos and materials

At home, Lami and his father engage in one activity in which one of them rolls the dice. Each of them then places the number of plastic eggs, as determined by the dice roll, into their own egg carton in any arrangement they choose. Afterwards, they compare their different arrangements and discuss these, including possible structures. Figure 1 provides a guideline for referring to the position of each egg in the text, as well as the arrangement made by Lami and his father for a dice roll of six.



Guideline for describing the positions (P) of the eggs in the arrangement

Lami's first arrangement of six eggs

Father's first arrangement of six eggs

Figure 1: Guideline for describing positions and arrangements

Repertoires-in-use by describing structures: Navigating additive and multiplicative language means

The analyzed scene starts when Lami rolls the dice again after he and his father have already played the game once. Lami initiates the scene by rolling a six, and he and his father then place the corresponding number of eggs into their respective egg cartons (see Figure 1). Upon seeing how his father has arranged the eggs, Lami exclaims: *"that's not six."*. From this moment, the scene centers on arguing why both arrangements consist of six eggs. In response to Lami's objection, the father encourages counting rather than arguing: *"isn't six? count, count."*. However, Lami does not follow this instruction. Instead, he attempts to engage in an argument by stating, *"look, you placed them differently."*. The father again requests to count. Lami counts the eggs one by one in the carton, but after completing the count and within the same turn, Lami reiterates his perspective, *"...but I placed them the same, because I won."*. The father seems unconvinced but does not ask Lami to clarify his point; instead, he rephrases it. The following analysis concentrates on the moment after this when Lami self-initiates another argument to support his reasoning: *"And look here, look."* (turn 101). It appears that Lami suddenly notices something else in the arrangement that supports his argumentation.

101 Lami: Y mira aquí, mira. (And look here, look.) *(puts the dice on the ground and turns it so that the dice face 6 is oriented as it might lie in the father's carton)* Es así. (It's like this.) *(points to the dots on the dice)* Tú los has puesto en (You placed them in) *(turns the dice so that the dice face 6 is oriented as it lies in his carton, touches the eggs in the father's carton in the order: P5, P3, P1)* uno, dos, tres. (one, two, three.) Y tres. (And three.) *(touches P7 – P9)* Aquí hay tres (Here are three) *(points to the top row P5, P3, P1)* y aquí (and here) *(points to squares P7-P9)* hay tres y es seis, pero (there are three and it's six, but) *(takes the egg from P5 and places it in P4)* si los pones (if you place them) *(takes the egg from P1 and places it in P2)* así, es igual (like this, it's the same) < *(places the eggs back as the father had placed them)*

102 Vater: Vale, vale, ¿y si los pongo así? (Okay, okay, and if I place them like this?) *(takes eggs from the bottom row and places them in squares P6, P8, P10, looks at the child)*

103 Lami: *(scratches his forehead, looks at the dice)* Queda dos, dos y dos. Tres, (It becomes two, two and two. Three,) *(shows three fingers)* tres dos. (three two.) *(looks at the father)*

Lami's utterance in turn 101 is remarkable, as it is highly condensed and much information has to be unfolded: He articulates *"You place them in one, two, three. And three..."*. In the utterance, Lami counts the eggs on the top row independently, he then bundles the eggs in the lower row, by subitizing (seeing and naming the set at one glance) not by counting each egg individually, and by collectively referring to them as *"... And three ..."* (see Figure 2). He subsequently rephrases and summarizes, *"Here are three and here there are three"*, and then addresses his father's earlier remark with, *"and it's six..."*. This clarification underlines that Lami's point is a different one: while the actual arrangement indeed consists of six eggs, it does not align with "his" game's rules, because it does not

resemble a dice showing six. However, Lami points out that the eggs in the carton can indeed be rearranged to visually represent the six on a dice and execute the rearrangement: *"but if you place them like this, it's the same."*. Lami's argumentation, referring to the arrangement of the points on the dice, remains primarily implicit and is supported mainly by gestures (see Figure 3).

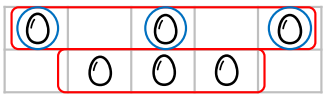


Figure 2: Lamis diagram in his father's arrangement

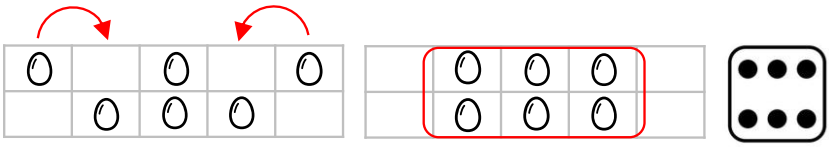


Figure 3: Lami's modification in his father's arrangement

The father's next move is a very productive one: *"Alright, alright, and if I put them like this?"* (turn 102). He shifts both eggs on the outside of the lower row one field outward (see Figure 4). Lami responds by describing the structure by unitizing the eggs with the phrase: *"It becomes two, two and two."* (see Figure 5).

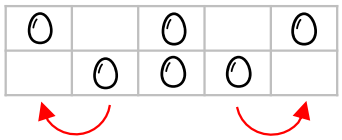


Figure 4: Father's changes in his own arrangement

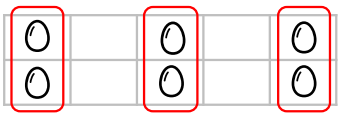


Figure 5: Lamis unitizing in groups of two

The lexical choice Lami makes in Spanish, "quedar" is particularly interesting. It can be interpreted as "it remains", "it leaves" or "it becomes". In this specific context, the use of the singular form "queda" rather than the plural "quedan" suggests a reference to the structure as a whole rather than to the individual eggs. This leads us to believe that Lami is verbalizing the structure as a complete whole, meaning not just subsets, three subsets of two, but as a whole of six that can be structured into groups of twos. This interpretation is emphasized by Lami's use of language in the second part of his turn: "tres dos" *(three twos)*. Here, Lami verbally stresses the number "three" not the size of each group, but the number of groups. His expression *"three two"* is condensed. In Spanish, the distinction between the size of the groups and the number of groups remains very implicit. To make it explicit, one would need to unfold the expression into "three groups of two". In this case, German, another language in Lami's repertoire, could be used productively to also highlight the bundle as "drei 2er" (three twos) German provides a very condensed means to express this concept, effectively emphasizing the groupings in a condensed manner (Prediger, 2019).

104	Vater:	Dos, dos .. ¿Tres por dos cúanto es? ¿Tres veces dos cúanto es? (Two, two ..Three (multiplied) by two how much is it? Three times two how much is it?)
105	Lami:	No lo sé. (I don't know.) *(briefly looks back)*
106	Vater:	¿Tres veces dos cuánto es? ¿Dos, (Three times two, how much is it? Two, *(points to P5 and P10)* dos (two) *(points to P3 and P8)* y dos, (and two) *(points to P1 and P6)* cuánto es? how much is it?
107	Lami:	*(leans forward to see the father's carton)* seis (six) *(looks at him)*
108	Vater:	#Pah, (Pha,) *(laughs)* vamos, (come on,) *(takes eggs out of the carton)* tíralo otra vez. (throw it again.)

Lami's way of structuring and verbally addressing the arrangement as *"three twos"* might have correctly led the father to think about multiplication. The father automatically switches to a formal-related register, asking, *"three (multiplied) by two, how much is it?..."* (turn 104). However, he does not use the extended expression "multiplicado por" *(multiplied by)* but the abbreviated "por". This reduction to the preposition "by" alone, a typical way to denote multiplication in Spanish, fails to convey the conceptual core embedded in the highly technical expression of one multiplicand being multiplied by a multiplier. The use of the preposition "por" without a verb could lead to other interpretations, such as the everyday phrase "three eggs for the price of two.". The father seems quickly to identify that this formal-related register doesn't mean much to Lami and shifts the choice of his language means to something more meaning-related: *"...three times two, how much is it?"* (turn 104). However, this approach to expressing multiplication with the idea of "times", also seems not to resonate much with Lami, who responds with *"I don't know"* (turn 105). The father then in turn 106 unfolds *"three times two"* to *"two, two, and two"*, which seems to work, but in doing so, he only addresses an additive structure, while Lami, in his use of language, is actually addressing a more sophisticated multiplicative structure by referring to the groups as "three twos". Lami addresses the multiplicative structure but is not yet able to formally name it. Lami and his father are navigating through different language registers. The expression "two two and two" leads Lami to say the correct answer. The father seems pleased that Lami gives the right result to the operation: "six". Saying it so they conclude this sequence.

Figure 6 serves as summary and illustrates that while discussing the structure of the arrangement, Lami and his father primarily used Spanish, but employed different language registers within this language. Lami's father navigated between formal-related and everyday language, primarily switching between these registers without explicitly connecting them to each other or the material. Lami, on the other hand, did not use formal-related language as he has not yet learned it, but is developing conceptual understanding by handling the materials and discussing them. He articulates the structure using language that is more related to meaning, even addressing more complex structures. These serve as a foundation for understanding the construction of other concepts, such as multiplication.

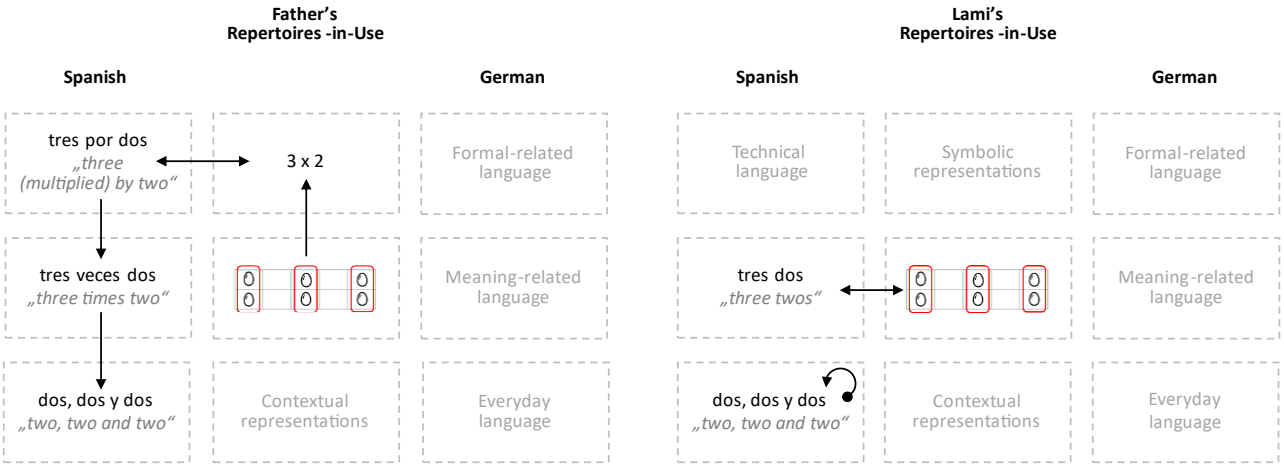


Figure 6: Summary of the repertoires-in-use of Lami and his father.

Discussion and Outlook

The empirical analysis of the presented case provides insights into the dynamics of early mathematics learning within a family setting and explores the diversity and complexity of linguistic repertoires-in-use. Lami and his father engage in an argumentation process, negotiating a common understanding of the meaning of "six" in a playful situation. The process reveals that the richness of language in mathematical interactions is not limited to the specific language spoken but encompasses the entire spectrum of an individual's linguistic and experiential repertoire. Additionally, it shows that within a single language, in this case Spanish, different uses of language can be reconstructed. The analysis highlights how the father's efforts to elicit counting from his son, possibly influenced by personal experiences in learning mathematics or cultural views, contrast with Lami's approach to playing the game and arguing in line with his own game rules. This underscores the importance of providing parents with guidance on how to facilitate the learning situation at home to shape mathematical understanding effectively, without discarding but rather integrating their own resources in a meaningful way.

The reconstruction of the repertoires-in-use shows that while the father uses a more formal-related register, the child uses language that is more related to meaning. The language used by both illustrates how the father, triggered by Lami's language use, identifies a multiplication in the arrangement but remains within an additive view of the situation. Meanwhile, Lami, although unable to formally name the multiplication, addresses the multiplicative structure by speaking about the arrangement. This situation clearly demonstrates how early mathematical experiences related to structural use are crucial for developing a foundation for the later understanding of concepts like multiplication.

Moving forward, further analysis of the project data should examine how these family dynamics and linguistic strategies might vary in other multilingual settings and across different levels of language proficiency.

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