Patterns of individual learning support with focus on sign activity and communicating about it: A comparison

Annika M. Wille, Barbara Ott

Patterns of individual learning support with focus on sign activity and communicating about it: A comparison

Annika M. Wille[1] and Barbara Ott[2]

[1] Leibniz Universität Hannover, Germany; wille@idmp.uni-hannover.de

[2] St. Gallen University of Teacher Education, Switzerland

Diagrammatic sign activity and communicating about it are central to mathematical activity and especially to learning mathematics. In this article, we use our analysis method, "Process Analysis of Mathematical Sign Activity and Communication about it" (PMSC), to examine episodes of individual learning support. The focus in this paper is on patterns that emerge in the PMSC analysis and on differences in different expert-learner pairs.

Keywords: Individual learning support, diagrammatic activity, interaction process, arithmetics.

Introduction

Learning mathematics can be understood in terms of Sfard (2001) as increasingly participating in mathematical discourse. According to Dörfler (2006), this also encompasses progressive participation in the social praxis of diagrammatic activities. Interaction with experts can be seen as an essential part of this process (Tiedemann, 2012). For children with difficulties in learning mathematics interaction with experts in individual learning support is especially important. Such an individual learning support can be understood as a specific process of interaction that is successively established by expert and learner through a mutually interrelated interpretation and action (Tiedemann, 2012). Tiedemann (2012) describes interaction patterns like the invitation-reply-evaluation pattern (Mehan, 1979), which can arise in situations of individual learning support. Even though such interaction patterns are usually unconscious to the participants of the interaction, they are significant for the learning or non-learning of the students. Accordingly, regarding interaction patterns, Bauersfeld (1980) speaks of "hidden dimensions" (p. 23) of learning situations. So far, little is known about whether, and if so, how, interaction patterns regarding diagrammatic activity emerge in individual learning support. At the same time, it is important to make preservice teachers aware of such hidden dimensions so that they can develop alternatives of action if necessary (Mason, 1987). The aim of our research is to identify possible interaction patterns regarding diagrammatic activity and communication about it in individual learning support in pairs of preservice teachers and students as expert-learner pairs. In this paper, we consider patterns that allow us to compare different pairs. By means of two pairs, we present the patterns to be observed.

Theoretical framework

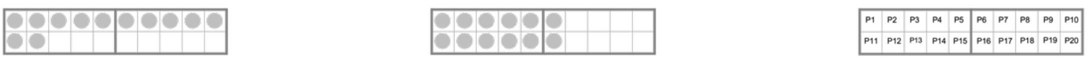
The term diagram is defined in various ways. In the following, diagrams are understood in the sense of the American philosopher Charles Sanders Peirce and are regarded as objects of mathematical activity (Dörfler, 2006). In this sense, diagrams are signs with a relational character, which are created based on the rules of a *sign system* and which can be operated and experimented with according to this sign system (Dörfler, 2006; Hoffmann, 2007). Activities with diagrams such as experimenting or observing are referred to in what follows as *diagrammatic activities* (Wille, 2020). They can give rise to mathematical concepts and understanding (Dörfler, 2006). For example, the natural numbers are such a sign system. Also, materials developed for mathematical learning processes can be considered diagrams. For example, the field of twenty with chips, which is used in the situations analyzed in this paper, is specially designed for mathematical learning processes and is subject to a preconceived system of rules (Vogel & Huth, 2020, p. 221), within which diagrammatic activities are possible. For students, each sign system is its own learning content, as is each direction in moving between different sign systems. Note that whether a diagram is understood as such depends on the observer who is familiar with a corresponding sign system to which it belongs. Another essential part of mathematical activity is communication about it (Dörfler, 2006). Communication about diagrams and diagrammatic activities involves both linguistic and gestural utterances (Huth, 2022).

Research Interest

Our research focuses on how diagrammatic activity and communicating about it in individual learning support are intertwined and occur in different sign systems. For this purpose, we developed the Process Analysis of Mathematical Sign Activity and Communication about it (PMSC) (Ott & Wille, 2022). In this paper, we ask: (RQ 1) What patterns can be identified by the PMSC? and (RQ2) How do differences in patterns emerge across pairs of preservice teachers and students as expert-learner pairs?

Setting

The episodes of individual learning support were carried out at the St. Gallen University of Teacher Education (St. Gallen, Switzerland), where preservice teachers support first- and second-grade children (ages 6 to 8) as part of an elective course. The sessions were videotaped. Activities of the project MALKA (Wehren-Müller et al., 2018) were implemented. The aim of the support is to disengage counting strategies in arithmetic tasks, which can lead to difficulties in the further course of mathematical learning (Scherer & Moser Opitz, 2010). In the support sessions analyzed here, the students were repeatedly shown several number cards. In each case, a corresponding amount of chips was to be arranged in the field of twenty in the form of a row or a block (Scherer & Moser Opitz, 2010) (see Figure 1).



a b c

Figure 1: Row arrangement (a), block arrangement (b), transcript numbering (c) (Ott & Wille, 2022)

A total of five expert-learner pairs participated in the study. In the following, two expert-learner pairs are considered: Ms. Schwyzer with Paula and Ms. Maurer with Sascha. Ms. Schwyzer is in the third, Ms. Maurer in the second of her six semesters of study. Sascha is a student in the second half of first grade, Paula a student in the first half of second grade. Both pairs met a total of seven times. Here we analyze the episode in which they first work with the described activity. For Sascha and Ms. Maurer this is their first support situation, for Paula and Ms. Schwyzer the fifth.

Method

The purpose of the PMSC is to provide a means of analyzing diagrammatic activity and communicating about it in different sign systems. The creation of an analysis sheet for the PMSC is based on an analysis method developed by Wille (2020) for imagined dialogues and was adapted by Ott and Wille (2022) for dyadic interaction processes. Before PMSC, an interaction analysis (Krummheuer & Naujok, 1999) is performed to reconstruct the interaction process in detail. Afterwards, the PMSC is conducted by creating an analysis sheet according to the following rules (see Figure 2): For each sign system used, there is one column. In the episodes discussed here, the sign systems are "loose chips", "field of twenty", "natural numbers", and "number cards". Communication that cannot be assigned to one of the sign systems is noted in a column labeled "others". In addition, the following is noted for the student (blue) and the preservice teacher (red):

- "If a *diagram* is used in a turn, a *filled circle* is set in the column of the corresponding representational system. If *communication about* diagrams is used, a *dashed circle line* is set. If both take place, both are noted together. A star is used if an activity cannot be interpreted as a diagram or communicating about it.
- The filled circles or dashed circle lines are connected to each other by *solid lines* if a connection is made by *diagrammatic activities*. The line is *dashed* when the connection is made by *communicating about diagrams*. If both occur, both are noted together.
- If, in a turn, *diagrams of different representational systems correspond* with each other, they are connected by an *arrow*. The direction of the arrow indicates which representational system is used as the starting point.
- If *diagrams that have already occurred once occur again in exactly the same way*, they are connected by *two narrow lines*." (Ott & Wille, 2022)

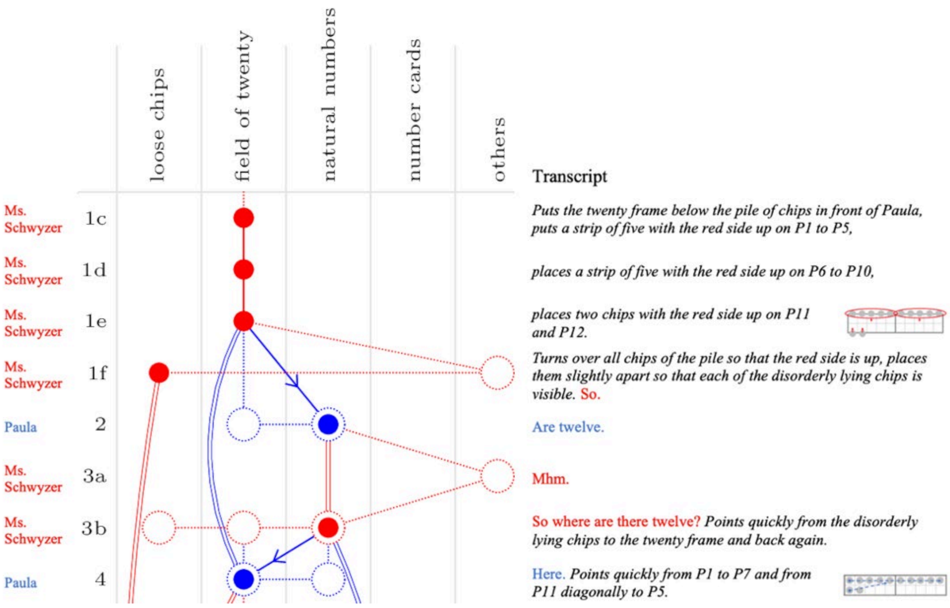


Figure 2: PMSC example, turn 1c to 4 of the episode with Ms. Schwyzer and Paula

Findings

Patterns in the PMSC

Regarding RQ 1, we distinguish *basic patterns*, *composed patterns* and *extended patterns*. *Basic patterns* consist of at least of two circles. We distinguish *transitions*, *connections* and *lines*. They may refer exclusively to diagrammatic sign activity or exclusively to communication about it. If both occur simultaneously, the dotted and solid lines are adjacent (see Table 1): *Transitions* occur when different sign systems correspond with each other, and one sign system is the starting point. They can take place over two turns or in one turn. In contrast to transitions, *connections* do not have a starting point and a destination point, but a simultaneity. *Lines* refer to only one sign system and are drawn vertically. They can consist of two or more connected circles.

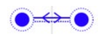
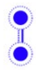
		
transition	connection	line

Table 1: PMSC basic patterns

Composed patterns are compositions of basic patterns. Various composed patterns are possible, the most common are mentioned here (see Table 2): *Boxes* are composed by connections or horizontal transitions and lines. As before, they can be diagrammatic, communicative, or mixed. Also, they can appear *one-sided* if it is diagrammatic only in one sign system. Boxes occur when utterances or diagrammatic activity refer to two sign systems at the same time. In one-sided boxes, the diagrammatic activity takes place only in one sign system, while at the same time the communication also refers to a second one. In addition, an *overlapping box* and *line* can be composed of lines that intersect with boxes, connections, or transitions.

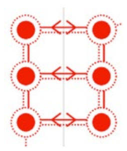
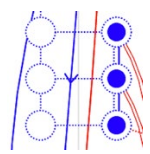
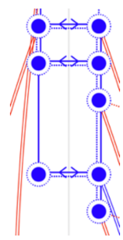
		
box	one-sided box	Overlapping box and line

Table 2: (Some) composed patterns

Finally, there are the extended patterns. They can be identified when a series of basic or composed patterns repeat each other exactly or closely. An example is shown when RQ 2 is discussed below.

Comparing the occurrence and interconnection of patterns

Regarding RQ 2 we will compare the occurrence and interconnections of patterns that emerge in the dyadic interaction processes of two pairs of preservice teacher and children: Ms. Schwyzer with Paula and Ms. Maurer with Sascha. Two differences will be discussed: First, references to diagrams or patterns and second, differences in extended patterns regarding the same mathematical task.

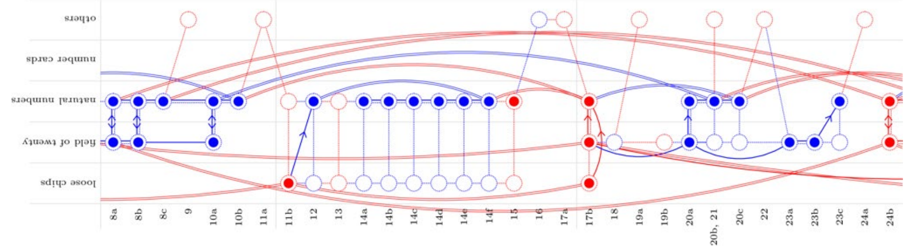


Figure 3: Turn 8a to 24b of Ms. Schwyzer's and Paula's PMSC

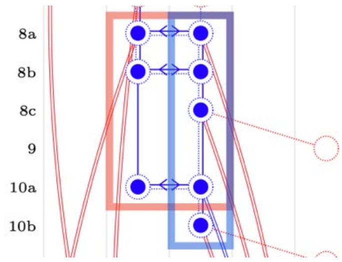
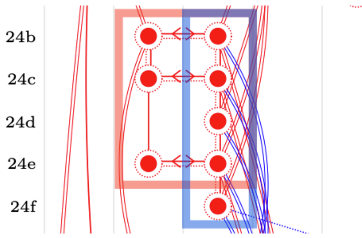
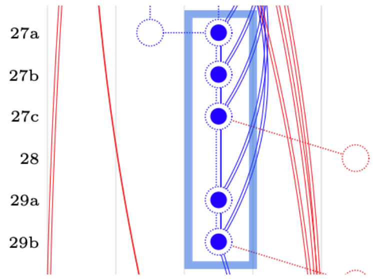
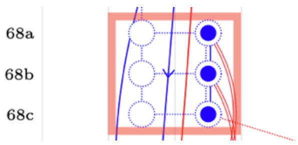
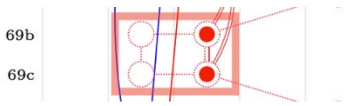
	The pattern of Paula	referenced by Ms. Schwyzer	referenced by Paula
a	 *Overlapping box and line*	 *Overlapping box and line*	 *Line*
b	 *One-sided box*	 *One-sided box*	

Table 3: References in the PMSC of Ms. Schwyzer and Paula

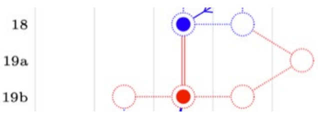
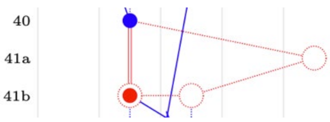
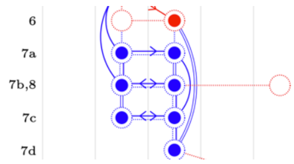
a	Diagrams of Sascha referenced by Ms. Maurer (turn 18 to 19b, turn 40 to 40b)  
b	A diagram of Ms. Maurer referenced by Sascha (turn 6 to 7d) 

Table 4: References in the PMSC of Ms. Maurer and Sascha

(1) When comparing the PMSC of the two pairs, it becomes apparent how differently the preceding composed pattern is taken up again. In the PMSC of Ms. Schwyzer and Paula, one can see many *double narrow lines* that denote if a diagram already occurred in the same way (cf. Figure 3). Hence, both Ms. Schwyzer and Paula often reference diagrams from before. But more importantly, the mode of references differs from those found in the PMSC of Ms. Maurer and Sascha. While in the latter only single diagrams are repeated, in the PMSC of Ms. Schwyzer and Paula diagrammatic activity and to some extent whole composed patterns are referenced. Examples can be found in Table 3a: Ms. Schwyzer (turn 24b to 24f) references an *overlapping box and line* from Paula (turn 8a to 10b). Afterwards, Paula references the *line*, hence the diagrammatic activity within the sign system natural numbers (turn 27a to 29b). Furthermore, in turn 69b to 69c one can see how Ms. Schwyzer references a *one-sided box* of Paula (turn 68a to 68c). In contrast, Ms. Maurer and Sascha exclusively reference individual diagrams. Examples are shown in Table 4, where Ms. Maurer takes up Sascha's diagrams (turn 18 and turn 19b or turn 40 and turn 41b) or Sascha takes up a diagram of Ms. Maurer (turn 6 and turn 8d).
(2) Another difference between the two pairs is the occurrence of extended patterns, where a number of basic or composed patterns repeat each other. There is an extended pattern that occurs in both pairs. A number of composed and basic patterns occur several times in the same order. However, this is less complex for Ms. Maurer and Sascha than for Ms. Schwyzer and Paula. This means that in the extended pattern of Ms. Schwyzer and Paula, the extended pattern is even more nested and more references occur. In the episodes of individual learning support, this means that some interaction patterns emerge in a similar way. However, the nesting in Ms. Schwyzer and Paula shows that they connect different sign systems to a greater extent than Ms. Maurer and Sascha and refer back to diagrams and the activity with them more often.

To sum up, Ms. Schwyzer and Paula repeat not only single diagrams, but often composed patterns and especially diagrammatic activity. Their extended pattern in arranging amounts of chips in the field of twenty is richer than Ms. Maurer's and Sascha's and Paula in contrast to Sascha makes changes back to the natural numbers.

Discussion

It turns out, that interaction patterns regarding diagrammatic activity emerge in individual learning support. Such interaction patterns show up both in the longer course in a support situation (extended patterns), as well as in very small sections (composed patterns). Some elements of the extended patterns seem to be conditioned by the activity: By repeatedly showing number cards and arranging the appropriate amount of chips on the field of twenty, some repetition is imposed. At the same time, however, these differences are also related to the social practices of diagrammatic activities established by different expert-learner pairs (Dörfler, 2006). As an example, Paula takes up the repetition of diagrammatic activity, whereas Sascha, just like Ms. Maurer, only repeats single diagrams. Furthermore, here, the multiple occurrences of lines in Ms. Schwyzer's and Paula's extended patterns are because they always arrange an amount of chips one after the other in the block and the row (or vice versa), while Ms. Maurer and Sascha always apply only one arrangement (block or row). Maybe these could be an indication that different goals pursued by the expert in the interaction process evoke different diagrammatic practices. At the same time, the pairs shown here worked on this activity at different times. It is possible that this also has an impact on diagrammatic practice. To address these questions, in subsequent research additional situations of the here shown expert-learner pairs (longitudinal comparison) and other expert-learner pairs with the here analyzed activity (cross-comparison) will be compared.

Acknowledgement

We thank Thomas Ries for programming the compiler to draw the analysis sheets of the PMSC.

References

Bauersfeld, H. (1980). Hidden dimensions in the so-called reality of a mathematics classroom. *Educational Studies in Mathematics*, *11*, 23-41. https://doi.org/10.1007/BF00369158

Dörfler, W. (2006). Diagramme und Mathematikunterricht [Diagrams and mathematics education]. *Journal für Mathematik-Didaktik*, *27*(3-4), 200-219. https://doi.org/10.1007/BF03339039

Hoffmann, M. H. G. (2007). *Cognitive conditions of diagrammatic reasoning*. Retrieved from http://works.bepress.com/michael_hoffmann/1/

Huth, M. (2022). Handmade diagrams – Learners doing math by using gestures. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education* (CERME12) (pp. 1543–1550). Free University of Bozen-Bolzano and ERME.

Krummheuer, G., & Naujok, N. (1999). *Grundlagen und Beispiele Interpretativer Unterrichtsforschung* [Basics and examples of interpretative teaching research]. Leske + Budrich.

Mason, J. (1987). Erziehung kann nur auf die Bewusstheit Einfluss nehmen [Education can only influence awareness], *mathematik lehren*, *21*, 4–5.

Mehan, H. (1979). *Learning lessons: Social organization in the classroom*. Harvard University Press.

Ott, B., & Wille, A. M. (2022). Diagrammatic activity and communicating about it in individual learning support: Patterns and dealing with errors. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education* (CERME12) (pp. 4312–4319). Free University of Bozen-Bolzano and ERME.

Scherer, P., & Moser Opitz, E. (2010). *Fördern im Mathematikunterricht der Primarstufe* [Support in primary mathematics education]. Spektrum.

Sfard, A. (2001). There is more to discourse than meets the ears: Looking at thinking as communicating to learns more about mathematical learning. *Educational Studies in Mathematics*, *46*, 13–57. https://doi.org/10.1023/A:1014097416157

Tiedemann, K. (2012). *Mathematik in der Familie* [Mathematics in the family]. Waxmann.

Vogel, R. & Huth, M. (2020). Modusschnittstellen in mathematischen Lernprozessen [Mode interfaces in mathematical learning]. In G. Kadunz (Ed.), *Zeichen und Sprache im*

Mathematikunterricht [Sign and language in mathematics teaching] (pp. 215–253). Springer Spektrum. https://doi.org/10.1007/978-3-662-61194-4

Wehren-Müller, M., Widmer, J., Hepberger, B., Moser Opitz, E., Ott, B. & Vogt, F. (2018). *Fördereinheiten für die individuelle Förderung*. [Support units for individual support]. Pädagogische Hochschule St. Gallen.

Wille, A. M. (2020). Activity with signs and speaking about it: Exploring students' mathematical lines of thought regarding the derivative. *International Journal of Science and Mathematics Education*, *18*, 1587–1611. https://doi.org/10.1007/s10763-019-10024-1