Perceiving and describing structures of kindergarten children

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Successful structuring processes are an important basis for arithmetical learning. This includes describing (individually) perceived structures of a set. Therefore, a learning situation with two groups of children in the last year of kindergarten (age 5) in Switzerland was analysed. In this paper we investigate which structures the kindergarten children perceived and described and whether they oriented themselves towards other participants in their descriptions. The results of the study show that children use four main strategies in their descriptions and that they adopt descriptions from other children as well as from the kindergarten teacher. These takeovers can be seen both positively (children learn how to describe structures) and critically, as they may inhibit their own individual perceptions.

Keywords: Perceiving structures, describing sets, early mathematics education, interaction.

Introduction

There is a broad discussion about teaching and learning in early childhood (Björklund et al., 2020). Numerous studies focus on which materials and structures can support children in their structuring processes, especially regarding patterns and structures. Finger patterns used to support a part-whole understanding (e.g., Kullberg et al., 2020) play just as much a role as well as, for example, dealing with repeating, growing or spatial patterns, e.g. by structuring a set with counters so that you can quickly see how many there are (e.g., Lüken & Kampmann, 2018). This paper analyses a learning situation where kindergarten children in their final year (age 5) are asked to arrange a set in a ten-frame structure (egg cartons) to quickly recognize the number. The analysis focuses on the described structures and participant interactions, discussing their significance for individual learning processes.

Theoretical and empirical background

Structuring processes

Numerous studies indicate a correlation between a structuring perception of sets and both arithmetical and general mathematical learning (e.g., Dornheim, 2008; Kullberg et al., 2020; Lüken & Kampmann, 2018). There is also a correlation between the development of mathematical number concepts and the use of different structuring approaches, which serves as a foundation for arithmetical learning (van Nes, 2009). Additionally, quickly recognizing structured quantities is correlated with later success in arithmetic lessons (Dornheim, 2008). Perceiving sets in structures is also a prerequisite for determining cardinality through structural use: only by perceiving a set in (sub-)structures it is possible to use these structures to determine the cardinality of the set (Sprenger & Benz, 2020a, b). When determining the cardinality of a set, two processes can be distinguished (Steffe & Cobb, 1988; Benz, 2014): the process of perceiving the set and the process of determining cardinality. Benz (2014) developed a theoretical model illustrating the connections between these processes. Initially created through an inductive approach, the model has been empirically evaluated (e.g., Benz, 2014; Sprenger

& Benz, 2020a, b). This model distinguishes three ways of perceiving a set: as individual elements, as a whole, or in (sub-)structures. Each way of perception offers different possibilities for determining cardinality (e.g., Benz, 2014; Sprenger & Benz, 2020a, b). Since perception is an invisible process, it is challenging to analyse and understand. Several research approaches exist to investigate the process of perceiving a set in (sub-)structures (see Sprenger & Benz, 2020b). One way is analysing eye-tracking data to gain insights into the invisible process of perception (Sprenger & Benz, 2020a, b). These analyses have led to the insight that children do not always say what they think. For instance, when children state they have counted the elements of a set, it does not necessarily mean they do not perceive structures (Sprenger & Benz, 2020a). Two other ways for inferring perceptual processes include asking for verbal explanations of perceptions or having children create their own set representations (e.g., Benz, 2014; Gervasoni, 2015; Lüken & Kampmann, 2018). For all three possibilities, interpretations must be cautious to make accurate statements about children's perception of (sub-)structures.

Learning in interaction processes

Mathematical learning in kindergarten is embedded into kindergarten socialization. Interactions between the learners can be seen as a constitutive element for learning (Krummheuer & Brandt, 2001). Such a collaborative learning (Langer-Osuna, 2018) in mathematics education works through interaction between children, where identity (this refers to the way in which children position themselves and others in social interactions), learning and participation are closely linked. Various forms of participation can be distinguished (Krummheuer & Brandt, 2001): A person can adopt another person's wording and apply it to a new content, or they can adopt the idea behind the content and reformulate it. Complete imitation (content and wording) is also possible.

Research interest

Perceiving and describing structures is an important prerequisite for arithmetical learning and it is therefore crucial for children, to learn reliable structures they can use for their calculation strategies (Kullberg et al., 2020). As the ten-frame is also a useful didactical presentation for representing such reliable structures in primary school, this structuring is ideal for investigating how children in kindergarten age perceive and describe structures. This leads to the first research question:

1. How do children in the last year of kindergarten describe sets within a ten-frame-structure?

Talking about structures can be challenging for children, especially those of kindergarten age (Sprenger & Benz, 2020b). This raises the question of whether interaction with others can have an influence on their own descriptions, which leads to the second research question:

2. How do children adopt descriptions of others during a learning situation?

Method

The data presented in this paper are part of the project "MATHEsprechen in kindergarten: Using language repertoires in mathematics productively right from the start", which investigates the overarching question of the extent to which the use of multilingual resources is stimulated in kindergarten children through mathematical activities with materials in a learning environment. The mathematical content of the learning environment is about perceiving and using structures in sets. The whole learning environment consists of five parts (three kindergarten parts, two family parts) and was tested in a design experiment in September 2023 in two Swiss kindergarten groups: the first with 10 children (average age: 5 years, 7 months), the second with six children (average age: 5 years, 3 months) in the last year of kindergarten. For the analysis presented in this paper, we examine the learning situation from part 1, conducted in German with both groups. The same teachers (KG1 and KG2) led both situations. KG1 is a person of the research team. Both kindergarten groups had the same learning situation in which they should find a possibility to put a number of eggs in the carton so that one can quickly see, how many eggs there are. The egg cartons for 10 eggs are familiar to the children from their everyday life and resemble a ten-frame, a key visual tool in primary school, whose basic structure (bundle of tens, structure of fives, etc.) can be extended to a field of twenty and hundred. The two learning situations were videotaped with parental consent, transcribed, and coded using MAXQDA, a qualitative data analysis software that helps researchers organize, code, and analyse qualitative data. In a circular, inductive process, four main categories were identified, and participant interactions were coded and analysed.

Results

Strategies for describing structures

Four main strategies were identified when the children placed and described the quantities four, five and six, which are presented below. The adoption of the descriptions of the different participants is described for each group. These relationships are indicated by arrows in the following figures.

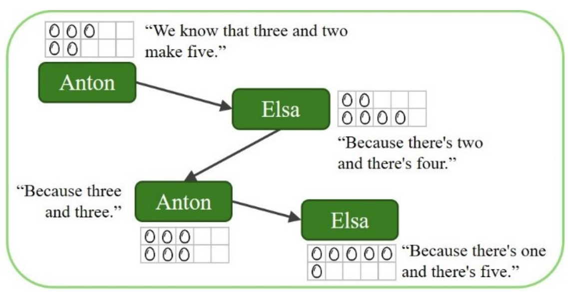

Figure 1: Adoption of the description for strategy "Describing subsets in rows" – (group 1)

In group 1 (green), Anton and Elsa each describe the quantities in two subsets. They name the number of eggs in each row, such as "We know that three and two make five" (Anton) or "Because three and three" (Anton) (see Figure 1). Elsa takes up this approach later and describes two more egg cartons using this strategy by describing the subsets of the two rows. This strategy was also observed in the second group (orange). Ester describes the egg carton with five eggs: "Because here are two and here are three" (see Figure 2). She first names the row with the smaller number of eggs and then the row with the larger number. Lilly adopts this approach immediately afterwards and also names the row with the smaller number first. Ester also keeps her strategy for other arrangements of the five eggs. She always names the smaller number of eggs first, regardless of whether this row is the top or the bottom row (see Figure 2). In an egg carton with one egg in the top row in the middle and three eggs in the bottom row (see Figure 2, right), Lami first describes the arrangement by counting the eggs one by one. Anna describes the set in two subsets: “Because here are three and there one” and Lami adopts this description and says: “There three and here one.”

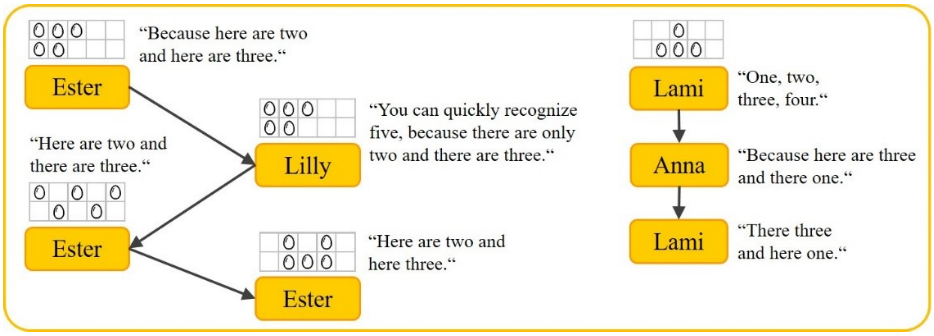

Figure 2: Adoption of the description for strategy “Describing subsets in rows” – (group 2)

Another strategy that children in both groups used to describe the quantities is to name subsets that do not correspond to the rows. In group 1 (see Figure 3) KG1 describes the set with five eggs: “I see four there and one more.” She points to the eggs on the left, which are arranged like a dice-four, then to the third individual egg in the top row. Naomi adopts exactly this description later (see Figure 3).

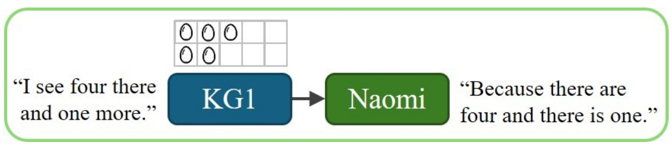

Figure 3: Adoption of the description for strategy “Describing subsets not in rows” – (group 1)

In the second group, Anna also describes this almost identical arrangement: “Because here are four and one” (see Figure 4). Lami takes up such a description in subsets and describes his arrangement: “Here is four and there is two.” He points with his hand to the four eggs on the left and the two eggs on the right (see Figure 4).

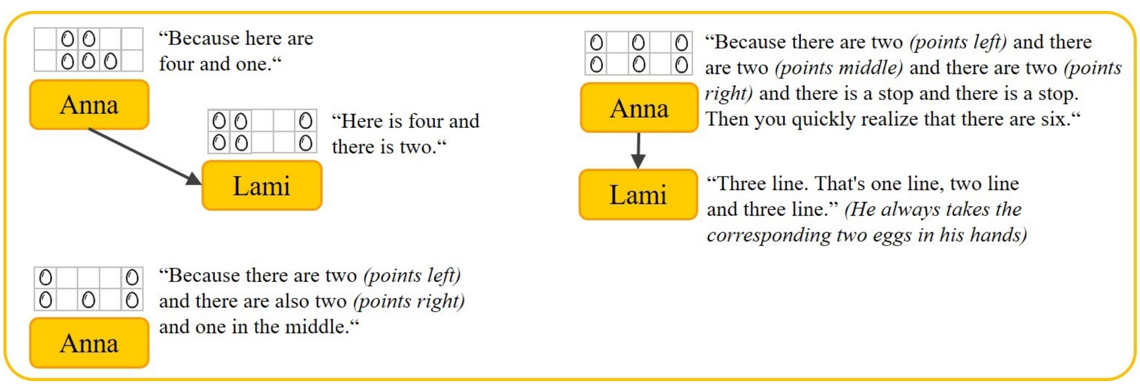

Figure 4: Adoption of the description for strategy “Describing subsets not in rows” – (group 2)

Anna arranges her eggs in three rows of two with the quantity of six eggs (see Figure 4). She points to a pair of two from left to right and explains: “...there are two and there are two and there are two...” Lami has initially sorted his eggs differently in his egg carton. He sees Anna’s arrangement and rearranges his eggs so that they resemble Anna’s arrangement. He also follows her example in the description and talks about three lines. He takes each of the two eggs in his hands and says “... That’s one line, two line and three line [*sic*].” The arrangement of the five eggs in the egg carton in Figure 3 above was placed by KG2. Anna names three subsets of why she can quickly see that there are five eggs: “Because there are two and there are also two and one in the middle” (see Figure 4, above).

A third strategy, particularly noticeable in group 1, is referring to empty places (see Figure 5 and 6). Right at the beginning of the learning situation, Elsa argues that there is an empty place in the egg carton with three eggs in the top row and two in the bottom row (see Figure 5). She says: “Only one is missing, then it would be six.” She is referring to the empty place under the third egg in the top row, which would create the picture of a dice-six.

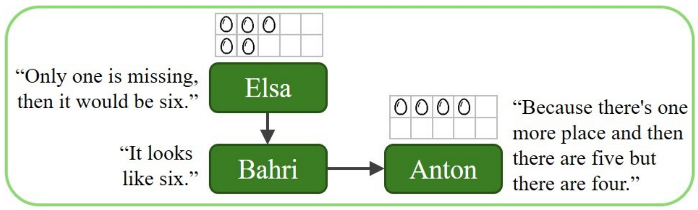

Figure 5: Adoption of the description for strategy “Referring to empty places” – (group 1)

Neither she nor Bahri, who takes up her idea and also says that it looks like a six in the same arrangement, explicitly names the dice pattern. At a later stage, Anton also explains that when four eggs are arranged in a row, it is easy to see that there are four because there is still a free place in the row (see Figure 5).

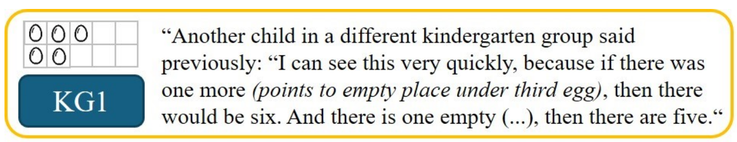

Figure 6: Adoption of the description for strategy “Referring to empty places” – (group 2)

In the other group, KG1 introduced this idea of referring to empty places in the conversation with the children. Throughout the learning situation, no child used this strategy. The fourth strategy observed was the explicit reference to dice patterns (see Figure 7 and 8).

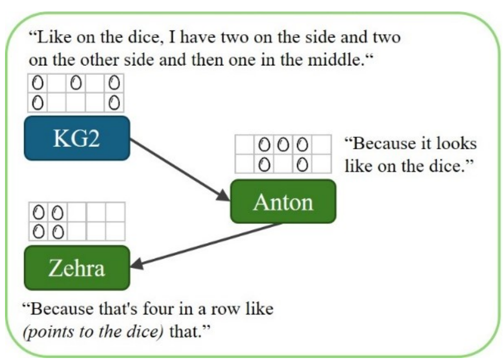

Figure 7: Adoption of the description for strategy “Referring to dice patterns (explicit)” – (group 1)

In the first group of children, KG2 has arranged the quantity of five eggs, which should correspond to the dice pattern of five. She describes: “Like on the dice, I have two on the side and two on the other side and then one in the middle.” Anton also describes a similar arrangement with a reference to the dice-five (see Figure 7). In the arrangement of the four eggs, Zehra refers to the dice-four as a reason why it is so easy to see that there are four eggs. KG2 adds both to Ester's and to Anna's description of the five eggs (see Figure 8) that it looks like a dice pattern.

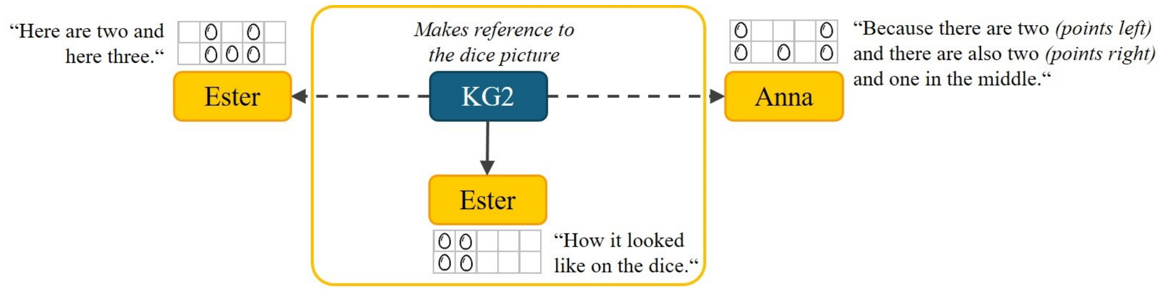

Figure 8: Adoption of the description for strategy "Referring to dice patterns (explicit)" – (group 2)

Only towards the end of the learning situation does Ester describe on her own initiative that the arrangement of the four eggs in the egg carton looks like on the dice. This situation is particularly interesting because a four was rolled with a large foam dice shortly beforehand and the dice pattern of four is still visible to her.

Adopting descriptions in interaction processes

As can already be seen in the previous results, the children refer to each other's descriptions in their descriptions. In the situation shown in Figure 3 there can be seen a complete imitation. Often the children adopt an idea of structuring (e.g. Describing subsets in rows), transfer this to a different arrangement of eggs, slightly (e.g. Figure 1) varying the wording. In Figure 5 Anton's reformulation of the idea to use the empty places differs greatly from Elsa's.

Lami, whose family language is Spanish and who is learning German in kindergarten, shows all forms of participation. In the situation shown in Figure 2, he first describes the arrangement of the eggs in the egg carton by tapping each one individually with his finger and counting them. Anna then explains that she recognises three (bottom row) and one (top row). Lami picks up on this explanation and names the same two subsets, pointing to the corresponding row at the same time (imitation). When laying the number of six eggs (see Figure 4, figures on the right), Lami shows another form of participation: He has initially arranged the eggs differently in his carton and then rearranges them to match Anna's arrangement (three groups of two). He then adopts the idea of Anna's description ("two and two and two") and reformulates it as "Three line [*sic*]. That's one line, two line and three line [*sic*]." He emphasises his explanation by raising each of the two eggs with his hands.

Different observations are made with respect of the role of the two KG in the participation. In group 1, the reference to the dice patterns was made by KG2 and this idea is taken up by Anton and Zehra (see Figure 7). Anton refers to almost the same representation as KG2 (dice-pattern of five), Zehra refers to the dice-pattern of four (see Figure 7) and reformulates the description of KG2. A naming of subsets that do not refer to the rows is initiated by KG1 in group 1 and is adopted by Naomi with the same wording (see Figure 3). In group 2, KG1 introduces a reference to empty places. However, this strategy is not taken up by any of the children (see Figure 6).

Discussion

The children in both kindergarten groups show a variety of strategies for describing quantities. Four main strategies can be identified: (1) Describing subsets in rows (see Figure 1 and 2), (2) Describing subsets not in rows (see Figure 3 and 4), (3) Referring to empty places (see Figure 5 and 6), (4) Referring to dice patterns (explicit) (see Figure 7 and 8). These descriptions allow assumptions to be made about the structuring perception of sets by different children. For example, in the case of Lami, it could be deduced that he does not perceive the set as subsets, but as individual elements (Sprenger & Benz, 2020a, b), since he is counting the eggs (Figure 2). This analysis changes, however, if we consider that Lami has taken over the arrangement of the eggs from Anna and that he initially arranged the four eggs differently in his egg carton. He does not succeed in interpreting a structure in this "foreign" arrangement and possibly uses counting as a familiar strategy (Sprenger & Benz, 2020a) to show that the eggs are the same in both cartons. This gives him the opportunity to "see" her individual perception of a structure and possibly adapt. This can also be seen in the exchange between Ester and Lilly (see Figure 2). After adopting Ester's description of subsets, Lilly ads: "Yes, or you count." As in the scene with Lami, it remains unclear whether we can say anything about Lilly's individual perception at this point, or whether she is merely repeating Ester's description, but counting would actually have been an equivalent or closer strategy for her (Sprenger & Benz, 2020a).

The results show that the children orientate themselves towards each other and adopt both, the idea or the wording of descriptions of structures (Krummheuer & Brandt, 2001). In this setting, children often take one strategy for describing quantities and apply it to different arrangements of eggs, typically adhering closely to the wording of their peers with slight modifications. This demonstrates how children possibly learn through social interaction and how they describe arrangements of quantities. This result must be interpreted with caution, as the children's prior knowledge was not measured in the study. Consequently, no assertion can be made regarding the children's extant structuring skills prior to the study, nor the influence these exerted on the observed interactions. However, the response to impulses from the KG varied between the two groups, indicating that a teacher's introduction of an impulse does not automatically lead to children adopting that description.

Learning from each other by adopting the wording and the strategy of describing quantities seems to play an important role in learning flexible structural descriptions of a set. This adaptation can certainly be viewed critically: It is positive that children learn how structures can be described. On the other hand, it is difficult to evaluate whether the child really perceives the set in this way, or whether individual perception is left "by the wayside". It is therefore important how the conversation is conducted in such a learning situation, what language is used, what questions are asked or what materials are offered. Such variety gives the child the opportunity to show what structures he or she perceives in the set. Careful planning is an important prerequisite, especially as children do not always say what they think (Sprenger & Benz, 2020a). Learning situations in groups seem to be very fruitful here, as children can learn from each other.

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